Decision Analytics in R

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Detecting rational bubbles in the Standard and Poor's 500 composite index using the present-value approach and cointegration technique

– SEMINAR PAPER –

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Contents

Abstract

The first decade of the 21st century has been stricken by two tremendous financial crises, the Dotcom bubble and the Financial Crises of 2007. Thus, identifying rational bubbles is extremely relevant for investors to prevent high losses. This paper gives a first outline for studying rational bubbles. It outlines two popular standard bubble tests, the variance bound test by Shiller and the Diba and Grossman test. The former is based on the present-value approach derived from fundamentals, the latter uses the technique of cointegration. Our finding suggests the time series under investigation, the Standard and Poor's 500 composite index, is driven by rational bubbles.

1 Introduction

The financial world is often distressed by financial crises. Every financial meltdown comes along with high losses for investors. To avoid potential profit setback, investors seek to identify speculative bubbles and accurately timed cut backs of investments. The most recognized crises in history of the Standard and Poor's composite index are shown in Figure 1. The Panic of 1907 is highlighted at first, followed by the Great Depression in 1929. After the World War Two, the Black Friday in 1987 is of historical interest. At the beginning of the 21st century, the Dotcom bubble was highly discussed; today, the Financial Crisis of 2007 is still present in everyone's mind.

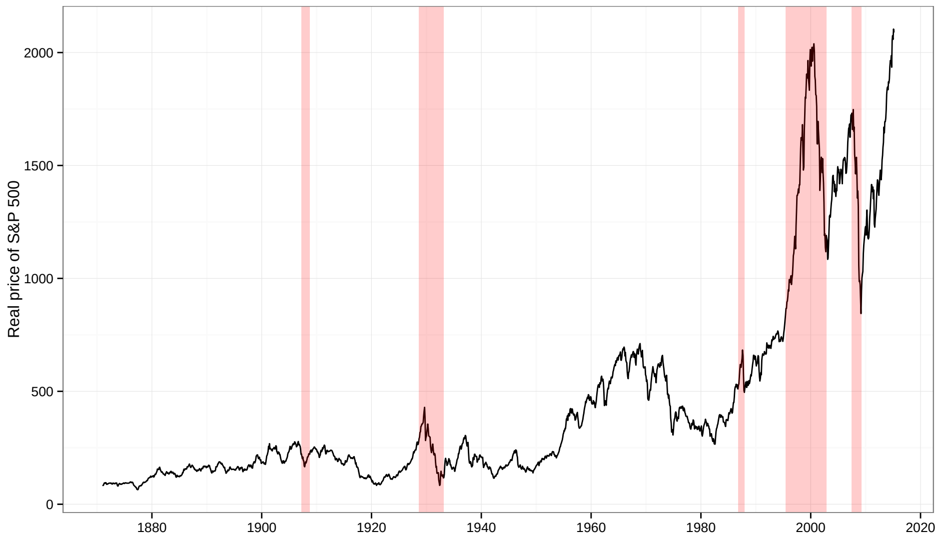

Figure 1: Monthly Standard and Poor's 500 real price composite index from January 1871 until March 2015 and its most important crisis.

Defining and detecting bubbles is crucial and a lot of effort has been spend to solve this problem. In recent literature there are several attempts to explain bubbles. Shiller [16] provides the explanation,*"by a bubble, some seem to mean any period when asset prices rise and then fall"*. Blanchard and Watson [1] specify a bubble as follows, *"rationality of both behavior and of expectations often does not imply that the price of an asset be equal to its fundamental value. In other words, there can be rational deviations of the price from this value, rational bubbles"*. In this paper, we use the terminology of a rational bubble in regard to [1].

The aim of this paper is to give an overview of the basic empirical test to identify rational bubbles. By applying the variance bound test from Shiller [17] and the Diba and Grossman [3], we answer the following research questions.

Research Question 1: Is the Standard and Poor's composite index driven by rational bubbles?

Research Question 2: Does the real price of the Standard and Poor's composite index and its corresponding real dividend payments follow a mutually long-run equilibrium path?

Our finding suggest the S&P 500 is, indeed, driven by rational bubbles. Additionally, we could not identify a long-run relationship between the real price of the S&P 500 and its corresponding real dividend payments, which is an indicator for rational bubbles. Alltogether, both tests indicate the existence of rational bubbles.
The rest of the paper is structured as follows. In Section 2, we give an overview of the methods used to identify rational bubbles. In Section 3, we present the empirical results of the variance bound test and the Diba and Grossman test. Finally, in Section 4, we critically discuss our main findings and provide an outlook for further research.

2 Methods

In this section, we start with required properties of time series, which is crucial when implemting the both tests. First, we present one property for a time series is stationary. Secong, the cointegration properties are outlined, to identify potential long-run relationships of the two time series in study. Afterwards, we empirical implement the Shiller test and the Diba and Grossman test.

2.1 Required Properties

Working with time series models has special prerequisites in terms of properties of the time series used. In our case, we need stationary time series. We pursue a more detailed approach, as we will first present the terminology of a *random-walk* process with *white noise* error term. Second, we explain the terminology of an integrated stochastic process and, third, define stationarity and cointegration.

2.1.1 Stochastic Process

To better understand the properties, we start with some considerations about stochastic processes. First, we assume that the current value of a variable Y_t depends on its value from the preceding period given by

$$y_t = y_{t-1} + \varepsilon_t \tag{1}$$

with the white noise term $\varepsilon_t \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$. Thus, the current price may be written as

$$y_t = y_0 + \sum_{i=1}^{t} \varepsilon_t. \tag{2}$$

The time series depends on its starting value y_0 and its aggregated white noise process, the stochastic trend. If time series follow a deterministic time trend, then a representation of the process y_t is

$$y_t = y_0 + \mu_0 t + \sum_{i=1}^{t} \varepsilon_t. \tag{3}$$

Regarding this notation , the process y_t follows two trends, a deterministic time trend $\mu_0 t$, which increases over time t and a stochastic trend $\sum_{i=1}^{t} \varepsilon_t$. This representation is useful, since it is important to distinguish between trend-stationary processes and non-stationary processes [4].

2.1.2 Stationarity

In a nutshell, stationary time series are charactrize by a constant expected value of a stochastic process y_t and a constant variance $V(y_t)$ at every point in time. A formal definition of stationarity is given in [11].
A stochastic process $\{y_t\}$, is called *stationary* if

$$E(y_t) = \mu \quad \forall t \in T, \tag{4}$$

$$E\left[(y_t - \mu_y)(y_{t-h} - \mu_y)\right] = \gamma_h \quad \forall t \in T \text{ and } h \in \mathbb{N} \quad \text{s.t.} \quad t - h \in T. \tag{5}$$

Econometric time series are often non-stationary, due to, among other things, seasonal or technological changes [8]. Additionally, time series may follow a trend-stationary process. Trend-stationary time series are stationary when accounting for a time trend. One common example is the gross domestic product (GDP). It is possible to transform a non-stationary time series into a stationary time series by using the first-difference $\Delta y = y_t - y_{t-1}$. Taking the first-difference is often sufficient to translate the non-stationary time series into stationary time series. But exceptions exist, for which one has to use higher differences. In general, one can say a time series y_t is integrated of order d, $y_t \sim I(d)$, if it must be differenced d-times to achieve a stationary time series, $\Delta^d y \sim I(0)$ [4].

2.2 Unit Root Test

One can conduct the Dickey Fuller test to check for stationary time series [11]. As mentioned before, it is highly relevant to distinguish between stationary and trend-stationary time series. Several testing specifications are possible. Note, we specified a stochastic process as $y_t = y_{t-1} + \varepsilon_t$. Such a unit root process allows us to run the following test procedure, in order to test for unit root via

$$y_t = \Phi y_{t-1} + \varepsilon_t \tag{6}$$

$$\Leftrightarrow y_t - \Phi y_{t-1} = \varepsilon_t \tag{7}$$

$$\Rightarrow \Delta y_t = \varepsilon_t \quad \text{with } \Phi = 1. \tag{8}$$

From Equation (8) one can derive the hypothesis test

$$\Delta y_t = \varphi y_{t-1} + \varepsilon_t, \tag{9}$$

with the null hypothesis H_0: $\varphi = 0$ and alternative hypothesis H_1: $\varphi > 0$. Under the null hypothesis, the process y_t is non-stationary.

Since testing for unit root is always the beginning of a comprehensive data analysis, one should avoid potential misspecification of the time series. Consequently, it is appropriate to first check the graphs of each time series and to decide afterwards whether or not to estimate the Dickey Fuller test with a constant, a linear time trend or both. This might result in testing the following stochastic processes, a random walk with a drift,

$$\Delta y_t = \mu_0 + \varphi y_{t-1} + \varepsilon_t, \tag{10}$$

or a random walk with drift and linear trend

$$\Delta y_t = \mu_0 + \mu_1 t + \varphi y_{t-1} + \varepsilon_t, \tag{11}$$

to be stationary.

Sometimes, it makes sense to include lags to avoid autocorrelation; then, the augmented Dickey Fuller (ADF) test is the right choice. The ADF test additionally includes autoregressive differenced terms for autocorrelation,

$$\Delta y_t = \mu + \varphi y_{t-1} + \sum_{i=2}^{p} \beta_i \Delta y_{t-i+1} + \varepsilon_t, \tag{12}$$

with p denoted as lags. However, the critical values for the augmented Dickey Fuller test do not follow a t-distribution, but valid critical values can be found in [12].

2.3 Cointegration

Cointegration refers to the subject of identifying a long-run relationship between two (or more) explanatory variables. Generally speaking, one wants to find an equilibrium relationship. For instance, values of time series will always fall back on track, after a short-time deviation from the long-run equilibrium value. In accordance to [7] one explanation of cointegration is as follows. Assume two variables x and y are integrated of order one, $x \sim I(1)$ and $y \sim I(1)$, then x and y are cointegrated, if a stationary linear combination, $y - \beta_1 x - \beta_0 \sim I(0)$, with $\beta_0, \beta_1 \in \mathbb{R}$ can be found. Thus, for cointegrated $I(0)$-variables $y - \beta_2 x \sim I(0)$ hold, such that the relationship

$$y_t = \beta_0 + \beta_1 x_t + u_t, \tag{13}$$

with $u_t \sim I(0)$ or u_t white noise exists.

2.3.1 The Engle and Granger Methodology

The Engle and Granger method [5] is a two-part procedure for two variables to identify a potential cointegration relationship. First, the variables are tested to be integrated of the same order. Second, if they are integrated of the same order, whether the linear combination of both variables are stationary or not. Because the two parts including several steps, we give a brief implementation guidance.

- Step 1: Test all variables for their order of integration using the Dickey Fuller test or the augmented Dickey Fuller test
- Step 2: If all variables are integrated of the same order, continue with estimating the long-run relationship

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t, \quad (14)$$

$$x_t = \beta_0 + \beta_1 y_t + \varepsilon_t. \quad (15)$$

Furthermore, the fitted residuals $\hat{\varepsilon}_t$ have to be checked for unit root via

$$\Delta\hat{\varepsilon}_t = d_1\hat{\varepsilon}_{t-1} + e_t, \quad (16)$$

with the null hypothesis, $H_0 : d_1 = 0$. If the null cannot be rejected, the residuals series is non-stationary and x_t and y_t are not cointegrated.

- Step 3: If the variables are cointegrated, a general error correction model (ECM) can be estimated to find out about the long-run behavior of the system. The ECM system is

$$\Delta y_t = \psi_0 + \gamma_1\hat{z}_{t-1} + \sum_{i=1}^{K}\psi_{1,i}\Delta x_{t-i} + \sum_{i=1}^{K}\psi_{2,i}\Delta y_{t-i} + \varepsilon_{1,t}, \quad (17)$$

$$\Delta x_t = \xi_0 + \gamma_2\hat{z}_{t-1} + \sum_{i=1}^{K}\xi_{1,i}\Delta x_{t-i} + \sum_{i=1}^{K}\xi_{2,i}\Delta y_{t-i} + \varepsilon_{2,t}, \quad (18)$$

where $\hat{z}_t$ is the error from the regression in Equation (16) and $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are the white noise processes. The ECM representation in Equation (17) declares the changes in y_t to its past changes, the past changes in x_t, and the previous period error $\hat{z}_{t-1}$ from the long-run equilibrium. For the cointegration analysis, the speed of adjustment coefficient γ_1 respectively γ_2 is of most interest, given that the coefficient determines the long-run adjustment. The coefficient should have a negative sign. If it does not have a negative sign, the system would not reach its long-run equilibrium path [14]. If the adjustment coefficient is not significantly different from zero, the variables do not respond to deviations from its long-run equilibrium path.

- Step 4: Asses model adequacy and perform diagnostics.

2.4 Testing for Rational Bubbles

In this section, we present two traditional tests to identify rational bubbles. First, we start with the variance bounds test for equity prices by [17], which is based on the classical present-value approach. Second, we present the Diba and Grossman [3] test to identify rational bubbles, which is based on a cointegration analysis.

2.4.1 Variance Bounds Test

Variance Bounds tests for equity prices goes back to [17] and [10]. The test was first used to test for the validity of the present-value approach [17] and later to identify rational bubbles [1, 20]. The Variance Bound test explain the existence of rational bubbles in the deviation of the real equity price from its expected equity price. The expected equity price is extrapolate from the fundamental value, hereafter present-value, of the asset. In detail, the present-value can be derived from expected future dividend payments. The underlying concept rest on the asssumption of equality between the real price of an asset and the assets expected future payments. By way of finding out about rational bubbles, one can calculate an upper bound of the variance based on fundamentals. Afterwards, this bound can be compared to the real variance of the underlying asset. Any violation of estimated bounds can be seen as the existence of rational bubbles [1, 20]. Now, we turn to theoretical considerations of the variance bounds test, where we follow the explanation of [6]. Concerning the present-value approach, the price of an asset P_t at time t can be calculated via

$$P_t = \sum_{i=1}^{\infty} \left(\frac{1}{1+r} \right)^i E_t \left(d_{t+i} \right), \tag{19}$$

with i is the future period, r is the discount factor, and $E_t \left(d_{t+i} \right)$ is the expected future dividend at time $t+i$. Thus, the *ex post* rational price is

$$P_t^* = \sum_{i=1}^{\infty} \left(\frac{1}{1+r} \right)^i d_{t+i}. \tag{20}$$

Assuming rational expectations, the difference between actual and expected dividends is a non-forecastable variable ε, with $E(\varepsilon) = 0$. Thus, the price can be write as

$$P_t^* = \sum_{i=1}^{\infty} \left(\frac{1}{1+r} \right)^i E_t \left[d_{t+i} + \varepsilon_i \right] = P_t + \sum_{i=1}^{\infty} \left(\frac{1}{1+r} \right)^i \varepsilon_{t+i}. \tag{21}$$

The upper bound on the variance can be found, if one considers the difference ε_t is uncorrelated with all information at time t. Thus, the variance of P_t^* can be written as

$$V\left(P_t^*\right) = \underbrace{V\left(P_t\right) + \varphi V\left(\varepsilon_t\right)}_{\geqslant V(P_t)}, \quad \text{where } \varphi = \frac{\left(\frac{1}{1+r}\right)^2}{1 - \left(\frac{1}{1+r}\right)^2}. \tag{22}$$

As mentioned in the beginning, a violation of this bound implies the price of the underlying asset cannot follow Equation (19), which may then be a result of existing bubbles [1, 20].

2.4.2 Diba and Grossman

Identifying rational bubbles through the fundamental price of an asset is discussed controversially in literature [e.g. 9]. As a consequence, [3] developed an approach to identify bubbles based on equilibrium theory. Diba and Grossman [3] suppose the existence of an equilibrium relationship between the stock price and the development of the dividends paid for the underlying asset indicates no rational bubbles. They suppose the fundamental value of the stock price similar to the present-value approach but with a fundamental factor unobservable to the econometrician [6]. The fundamental value of the stock price in the model is

$$P_t^f = \sum_{i=1}^{\infty} \left(\frac{1}{1+r} \right)^i E_t \left(d_{t+i} + u_{t+i} \right), \tag{23}$$

where i is the future time period, r is the discount factor, $E_t(d_{t+i} + u_{t+i})$ is the expected dividend plus the unobserved fundamental factor u_{t+i}.

If one assume the same order of integration for future dividends d_{t+i} and the unobservable factor u_{t+i}, the fundamental value P_t^f and the future dividends d_{t+1} should have the same level of integration [15]. Thus, based on [3], [15] conclude with the following bubble test.

- A rational bubble *does not exist*, if dividends as well as the stock price are stationary after the n-th difference and both are cointegrated.
- A rational bubble *exists*, if dividends as well as the stock price are non-stationary after the same n-th difference.
- A rational bubble *exists*, if dividends as well as the stock price are stationary after the n-th difference, but both are not cointegrated.

If a cointegration relationship exists, the ECM of the Engle and Granger method can be estimated to find out about the long-run behavior of the system.

3 Results

In this section, we apply Shiller's Variance Bound test and the Diba & Grossman test to real data. First, we present a detailed description of the data. Subsequently, we perform some diagnostic test and finally, we conclude with our results.

3.1 Data

As data source we use the *Standard & Poor's 500 Composite Index* from January 1871 until March 2015 and its dividend payments during this period. The data is provided by Robert Shiller's website [19]. The data set includes monthly time series of the real price of the S&P 500 and its real dividend payments. Additionally, Shiller reports the long-run interest rate for each month. Many researcher are using the same data source for their empirical analysis which can be seen as an advantage. Consequently, one can easily compare different approaches and time periods. For a more detailed description of the data, see [18].

Table 1 presents the descriptive statistics. Unfortunately, the data does not seem to be perfectly normally distributed due to skewness and kurtosis. Normally distributed time series should have a skewness of 0 and a kurtosis of 3. Nevertheless, the values do not deviate to much and we will continue with our estimation.

Variable	Mean	Median	Min.	Max.	Std. Dev.	Skewness	Kurtosis
Real price	445.5	236.1	64.2	2104	453.95	1.84	2.51
Real dividend	13.76	11.79	4.731	40.99	6.84	1.00	0.79

Table 1: Descriptive statistics.

3.2 Findings of the Variance Bound Test

Figure 2 shows the Standard and Poor's (real detrended) monthly composite price index from January 1871 until March 2015 and its corresponding detrended perfect foresight series.

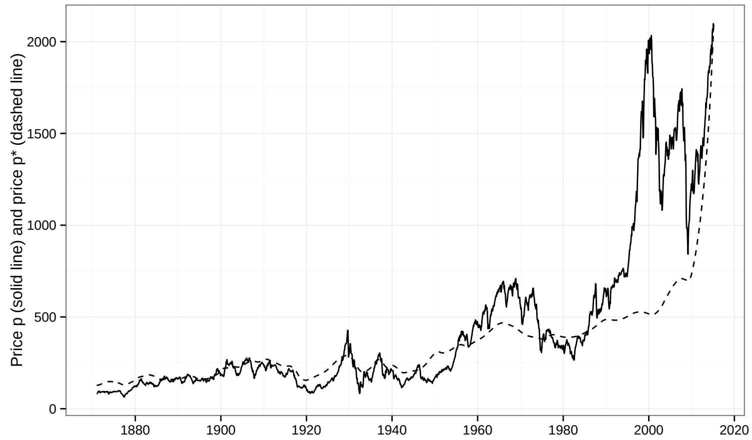

Figure 2: Monthly Standard and Poor's 500 real detrended price composite index from January 1871 until March 2015 (solid line) and its corresponding detrended perfect foresight series (dashed line).

The detrended perfect foresight series

$$p_t^* = \gamma \left(p_{t+1}^* + d_{t+1}\right), \qquad \text{with } \gamma = \frac{1+g}{1+r} \tag{24}$$

implied by Equation (20) is calculated recursively [17]. The constant growth rate g is the average growth rate of the real price over the entire sample size and the discount factor r is the average long-run interest rate over the entire sample size. In line with [17], we use the terminal value p_T as starting point which is the average detrended price

$$p_T = \frac{1}{T}\sum_{t=1}^{T} \frac{p_t}{1+g}, \tag{25}$$

over the entire sample size with T is the total number of month. The standard deviation of the detrended real price time series p_t is $\sigma_{p_t} = 452.72$ and the standard deviation of the perfect foresight series p_t^* is $\sigma_{p_t^*} = 234.22$. The corresponding variance is, $V_{p_t} = 204{,}959.40$ respectively $V_{p_t^*} = 54{,}858.33$, indicating the variance of the detrended real price is approximately four-times higher then the perfect foresight price. This result corresponds with Shiller's findings [17]. These findings indicates a violation of the variance bounds from Equation (22), hence there might be rational bubbles and investors should be aware, when taking investments in the S&P 500.

3.3 Findings of the Diba and Grossman Test

Next we present the cointegration analysis for the Diba and Grossman test. As mentioned in Section 2, we start with testing the time series for stationarity using the augmented Dickey Fuller (ADF) test. Before we apply the ADF test, we perform several information criterion tests in order to learn about the optimal length of lags. Subsequently, the Engle and Granger method is applied to find out about the cointegration relationships of the two underlying time series.

At the beginning, we need to decide upon the number of lags. To determine the optimal lag structure, we perform the Akaike Information Criterion (AIC), Schwarz Information Criterion (SC), Hannan-Quinn Information Criterion (HQ), and Final Prediction Error (FPE). Our findings suggest a lag length of three, seven or nine depending on the information criteria. Since there is not just one method but different views on how to determine lag length, we also perform the Portmanteau test for autocorrelation. The test helps us to set the optimal lag length [8]. None of the lag length indicating significantly improvements in the sense of less autocorrelation compared to the other. In the sense of degrees of freedom and model complexity it seems to be appropriate to choose the lowest lag length of three.

Determenistic Component	AIC(n)	HQ(n)	SC(n)	FPE(n)
none	9	7	3	9
constant	9	7	3	9
trend	9	7	3	9
both	9	7	3	9

Table 2: Information criterion tests with a maximal lag length of 12.

In the next step, we perform the ADF test. Figure 3 shows the time series. On the left hand side, the monthly data of the Standard and Poor's 500 real price composite index from January 1871 until March 2015 is shown and, on the right hand side, the corresponding dividend payments are shown during the same time period. From a first look, we can conclude that the real dividend payments follow a drift and the real price curve follow a drift and a trend. Therefore, we perform the ADF test with three lags and include a drift and a trend for the real price time series and only include a drift term, when testing the real dividend time series to be stationary.

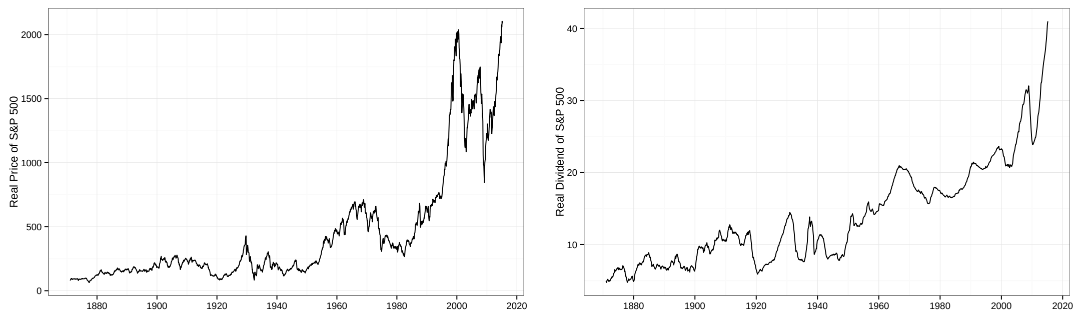

Figure 3: Monthly Standard and Poor's 500 real price composite index from January 1871 until March 2015 and its dividend payments in this time.

The results from the ADF tests are shown in Table 3. From Table 3, we can conclude that both time series are integrated of order one. This implies the time series are non-stationary in levels, but stationary in first differences. Hence, we continue with the next step and apply the cointegration test by Engle and Granger.

Variable	Deterministic Trend	Lags	Test Value	Critical Values		
				1%	5%	10%
p	constant, trend	3	0.00	−3.96	−3.41	−3.12
Δp	constant,trend	3	−18.22	−3.96	−3.41	−3.12
d	constant	3	−0.46	−3.43	−2.86	−2.57
Δd	constant	3	−10.67	−3.43	−2.86	−2.57

Table 3: Augmented Dickey-Fuller test for stationary indicating that all time series are stationary in first difference. Conducting further specifications by varying the lag length indicate the same results.

Performing Step 2 of the Engle and Granger method yields to the following two long-run relationships,

$$p_t = -364.387 + 58.842 d_t + \varepsilon_t, \qquad \text{with } \hat{\varepsilon}_t = 0.997 \hat{\varepsilon}_{t-1} + e_t \tag{26}$$

$$d_t = 7.813 + 0.013 p_t + \varepsilon_t, \qquad \text{with } \hat{\varepsilon}_t = 0.999 \hat{\varepsilon}_{t-1} + e_t \tag{27}$$

with highly significant p-values. Applying the ADF test to the residuals leads to the result that the residual time series is non-stationary and thus p_t and d_t are not cointegrated. However, the ADF test sometimes does not reject the null hypothesis (non-stationary) even if the time series is stationary. Therefore, in Figure 4 the residuals are shown, to find out about stationary.

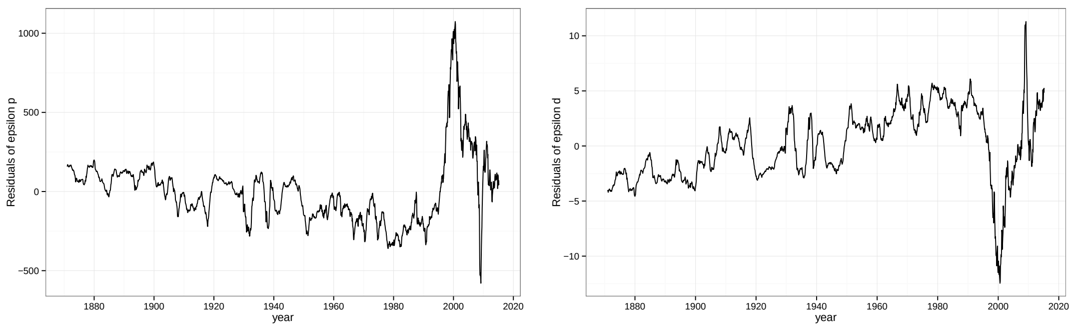

Figure 4: Left: Residuals of the Equation (26); right: Residuals of the Equation (27).

Strong fluctuation after the 1990s may explain why the residuals are non-stationary. This is also the result which the implementation of the Engle and Granger method in the software package R suggest. When running the given procedure in the egcm-package [13] find *"The series seem cointegrated but the residuals are not AR(1)"*. We proceed with further steps of the Engle and Granger method and do error correction introduced in Section 2.3.1. We perform the error correction model with three lags, such that the general representation of Equations (17) and (18) becomes,

$$\Delta p_t = \psi_0 + \gamma_1 \hat{z}_{t-1} + \sum_{i=1}^{3} \psi_{1,i} \Delta d_{t-i} + \sum_{i=1}^{3} \psi_{2,i} \Delta p_{t-i} + \varepsilon_{1,t}, \tag{28}$$

$$\Delta d_t = \xi_0 + \gamma_2 \hat{z}_{t-1} + \sum_{i=1}^{3} \xi_{1,i} \Delta p_{t-i} + \sum_{i=1}^{3} \xi_{2,i} \Delta d_{t-i} + \varepsilon_{2,t}. \tag{29}$$

The results of the error correction model estimation are presented in Table 4 and Table 5.

	Estimate	Std. Error	t-value	Pr(>|t|)	
ψ_0	0.7255	0.5378	1.35	0.1775	
γ_1	-0.0019	0.0026	-0.73	0.4655	
Δp_{-1}	0.2206	0.0241	9.16	0.0000	***
Δd_{-1}	-11.8549	4.5062	-2.63	0.0086	**
Δp_{-2}	-0.0444	0.0246	-1.81	0.0705	.
Δd_{-2}	6.4262	5.1399	1.25	0.2114	
Δp_{-3}	0.0541	0.0240	2.25	0.0243	*
Δd_{-3}	13.1481	4.5275	2.90	0.0037	**

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 22.06 on 1719 degrees of freedom
Multiple R-squared: 0.05947, Adjusted R-squared: 0.05564
F-statistic: 15.53 on 7 and 1719 DF, p-value: $< 2.2e-16$

Table 4: Error correction model of $p_t \sim d_t$.

	Estimate	Std. Error	t-value	Pr(>|t|)	
ξ_0	0.0049	0.0029	1.70	0.0886	.
γ_2	0.0009	0.0009	1.01	0.3125	
Δp_{-1}	0.0001	0.0001	0.65	0.5177	
Δd_{-1}	0.5586	0.0241	23.22	0.0000	***
Δp_{-2}	0.0001	0.0001	0.58	0.5646	
Δd_{-2}	0.1439	0.0275	5.24	0.0000	***
Δp_{-3}	-0.0001	0.0001	-0.46	0.6474	
Δd_{-3}	0.0313	0.0241	1.30	0.1946	

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.1179 on 1719 degrees of freedom
Multiple R-squared: 0.4715, Adjusted R-squared: 0.4693
F-statistic: 219.1 on 7 and 1719 DF, p-value: $< 2.2e-16$

Table 5: Error correction model of $d_t \sim p_t$.

We conclude that the speed of the adjustment coefficient is in both model specifications not different from zero and thus the system does not respond to long-run deviations. According to the outcome of Equations (26) and (27), this outcome is not a surprise. First, we could not clearly specify stationary residuals in Step 2 and as a consequence the ECM estimation stood on weak grounds since the very beginning. Second, a potential estimation error in Equations (26) and (27) transfers to the ECM which results in an even greater error. Finally, we could not find a definite cointegration relationship between the real price p_t and the dividend payments d_t. These findings indicate the existence of rational bubbles. In summary, both tests point to the existence of rational bubbles.

3.4 Discussion

This section discusses our results. We applied the variance bound test introduced by Shiller and the Diba and Grossman test to identify whether or not the S&P500 is driven by rational bubbles. The variance bound test shows the variance of the real price fluctuates four-times stronger than the expected price, estimated based on the present-value approach. Hence, we find approving evidence for our research question 1. The cointegration analysis could not indicate a long-run relationship between the real price of the S&P 500 and its corresponding dividend payments. According to the Diba and Grossman test, this may serve as evidence for the existence of rational bubbles. We may conclude that we cannot confirm research question 2. This result is in line with the result from the variance bound test, which shows evidence for rational bubbles.

However, these results should be treated with caution. First of all, as noticed in [6], any study that identifies a rational bubble is most likely counterbalanced by another study that does not identify a rational bubble as it keeps less strict assumptions on fundamentals. Moreover, the Schiller and the Diba and Grossman test, can only identify rational bubbles, but ignores behavioral aspects. Literature has produced a large body of research focussing on behavioral models. These models include behavioral indicators to detect irrational bubbles. Last but not least, the standard test implemented here, does not discover the presence of breaking points in a rational bubble. For this purpose, [2] developed a test to identify structural breaks, by testing the time series against a change in the order of integration for a particular point in time.

In sum, rational bubbles appear to exist and may have dire consequences thus investors should be aware of rational bubbles. Nevertheless, further investigation of rational bubbles for the S&P500 may be worthwhile, especially to find out about the exact beginning and ending of bubbles.

4 Conclusion and Outlook

Identifying rational bubbles is extremely relevant for investors to prevent high losses. The variance bound test by Shiller and the Diba and Grossman test is a good starting point to find out about rational bubbles in the underlying market. This paper gives a brief insight into the theoretical background of the two tests and tested the S&P 500 for the existence of rational bubbles.

Our findings suggest the S&P 500 is clearly driven by rational bubbles. The variance bound test, based on the present-value approach, shows movements of the S&P 500 clearly exceed the variance bounds given in the test. The cointegration analysis indicates no long-run equilibrium path for the real price of the S&P 500 and its corresponding dividend payments. Hence, the Diba and Grossman test suggest the same results as the variance bound test.

For future work, several research directions are of high interest. First, one might include other test procedures as mentioned in this paper. Second, beside knowledge on the existence of rational bubbles one should acquire knowledge on the break point of the bubbles. For such reason, one might investigate the time series of a structural change from stationary time series to non-stationary time series. Third, based on behavioral research, one might account for an information bias and incorporate news sentiment.

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