Question: Cartesian Product

a) Proof that for sets A and B the equation $A \times B = B \times A$ is not always true. 1

b) Let $A = \{0, 1, 2\}$, $B = \{2, 3\}$, $C = \{34\}$, $D = \{2, \{2, 3\}, \text{green}\}$ and $E = \emptyset$. 1
Calculate the following Cartesian products:

- $A \times B$,
- $C \times D \times E$, and
- $B \times C \times D$.

c) Proof $(A \cup B) \times C = (A \times C) \cup (B \times C)$ for all sets A, B and C. 1

d) Let A, B, C be sets. Proof $|(A \cup B) \times C| \leq |A \times B| + |B \times C|$. 1

Question: Powerset

1

a) Compute $\mathcal{P}(A)$ with $A = \left\{ 1, \{ a, b \}, \emptyset, 0 \right\}$. 1

b) Let A be a set. Proof $|\mathcal{P}(A)| = 2^{|A|}$.

Question: De Morgan's Law

1

a) Proof the second law of De Morgan and depict it visually in the form of a Venn diagram, i. e. $X \setminus (A \cap B) = (X \setminus A) \cup (X \setminus B)$ for a set X and subsets $A, B \subseteq X$.

Question: Boolean Operations

a) Proof the commutative law for the operator $\wedge$ in Boolean algebra. 1

b) List all possible unitary Boolean operations. 1

c) How many binary Boolean operations are possible? 1

Question: Normal Forms

1

a) Find the conjunctive normal form of f as defined through the truth table.

x	1	0	1	0	1	1	0	0
y	0	0	1	1	0	1	0	1
z	0	0	1	1	1	0	1	0
$f(x,y,z)$	0	1	1	0	1	0	1	0

1

b) Given the conjunctive normal form $(x \vee y) \wedge (x \vee \neg y)$, find

- an equivalent disjunctive normal form,
- an equivalent minimal form (simplify as much as possible),
- the full disjunctive normal form $(\bigvee_{i=1}^{n} \bigwedge_{j=1}^{n} (\neg)x_{ij})$.

1

c) Find the conjunctive normal form of the following expression:

$$(((x \vee y) \wedge (z \vee y)) \vee (z \wedge y)) \wedge \neg(y \vee (\neg z \wedge x)).$$

1

d) Find a disjunctive normal form of the negation of

$$(x \vee y \vee z) \wedge (x \vee y \vee \neg z) \wedge (x \vee \neg y \vee z).$$

Question: Derived Operations

a) Show: $(a \Rightarrow b) \wedge (b \Rightarrow a) \quad = \quad a \Leftrightarrow b$. 1

b) Show: $a \Rightarrow b \quad = \quad \neg b \Rightarrow \neg a$. 1

Question: Quantifiers

1

a) Negate the following propositions:

- $\exists$ key $\forall$ locks: The key fits into the lock.
- $\forall n \in \mathbb{N} \quad \exists x \in \mathbb{Q} \quad n = x^2$

Question: Regular Expressions

a) Visualize the expression `c+(ab|ba)` as an automaton. 1

b) Write a regular expression that captures German IBAN numbers. 1

c) Start with the following R statement: 1

```
str_vec <- c("173", "074", "432", "991", "132")
```

Use a regular expression that matches 173, 432 and 132. Find two different regular expressions to solve the task, but without using the operator `|`.

d) Consider the following string: 1

```
str <- paste0("Germany 0761 4231, +49177-234 123,",
              "Result, 1234567, 5654, 0160/44 22 123")
```

Use a regular expression to extract the three phone numbers listed here. Consider the different formatting symbols used and avoid matching the inccorect numbers 1234567 and 5654.

Hint: Use the `regmatches` function to extract the numbers given the results of `gregexpr`.