

## Elementary Concepts in Linear Algebra

### Number Representations

a) Convert 1 110 000 101 from base 2 into base 10 by hand.

1

b) Convert 852 from base 10 into base 2 by hand.

1

c) Repeat the above base 2 conversions by utilizing R.

3

d) Convert 756 001 from base 8 into base 10 by hand.

1

e) Convert 9530 from base 10 into base 8 by hand.

1

f) Repeat the above base 2 conversions by utilizing R.

3

g) Write a function `dec_to_bin` that takes a decimal number as input and outputs its binary representation in a vector of 1s and 0s. Instead of using the build-in conversion from R, implement this with help of an appropriate loop. Then test its correctness by calculating the binary representation of 852.

10

### Polynomials

h) Write a function `my_power` in R that calculates the exponentiation of two numbers  $b^n$  using a for-loop. Assume that  $n \in \mathbb{N}$ . Finally, use this function to calculate  $7^3$  and  $5^0$ .

4

i) Create a function `horner` that evaluates a polynomial at given point  $x_0 = 3$  by utilizing the Horner scheme for a given vector of polynomial coefficients. For example, the polynomial  $2x^4 - 4x^3 - 5x^2 + 7x + 11$  corresponds to a vector representation `c(2, -4, -5, 7, 11)` in R.

4

## Linear Algebra

2

- j) Calculate the dot product for the following two vectors by hand and with the help of R.

$$\mathbf{x} = \begin{bmatrix} 5 \\ -1 \\ 2 \end{bmatrix} \quad \text{and} \quad \mathbf{y} = \begin{bmatrix} 8 \\ 7 \\ -3 \end{bmatrix}.$$

2

- k) Write a function `dot_product` in R to calculate the dot product of the vectors from the previous exercise. Use a for loop to calculate the result.

2

- l) Write a function `mult_matrix_vector` in R to multiply a matrix by a vector. Use a for loop to calculate the result. Apply your function to  $A$  and  $\mathbf{x}$  defined by

$$A = \begin{bmatrix} 4 & 3 \\ -1 & 7 \\ 0 & 12 \end{bmatrix} \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

and then compare to the built-in functionality within R.

2

- m) Write a function `mult_matrix_matrix` in R to multiply two matrices. Use a for loop to calculate the result. Apply your function to  $A$  and  $B$  with

$$A = \begin{bmatrix} 4 & 3 \\ -1 & 7 \\ 0 & 12 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & 2 & 5 & 1 \\ -2 & 0 & 4 & 1 \end{bmatrix}.$$

Hint: The result should be  $AB = \begin{bmatrix} 6 & 8 & 32 & 7 \\ -17 & -2 & 23 & 6 \\ -24 & 0 & 48 & 12 \end{bmatrix}$ .

3

- n) Consider the vector

$$\mathbf{x} = \begin{bmatrix} -3 \\ -4 \\ -1 \\ 0 \\ 2 \end{bmatrix}.$$

What are its  $L^1$ -,  $L^2$  and 4-norm?

2

- o)** Calculate the inverse (or pseudoinverse, depending on what is necessary) for the following matrix. Double-check subsequently.

$$A = \begin{bmatrix} 3 & 7 \\ -1 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 8 \\ 3 & 12 \end{bmatrix}$$

1

- p)** Find the determinant, eigenvalues and eigenvectors to the matrix

$$A = \begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix}$$

by hand.

1

- q)** Calculate the determinates, eigenvalues and eigenvectors of the two matrices

$$A_1 = \begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad A_2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

with the help of R.

1

- r)** Are the following two matrices positive or negative definite?

$$A_1 = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} \quad \text{and} \quad A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$